Optimal Mating Strategies for Simultaneous Hermaphrodites in the Presence of Predators

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Two *Physa acuta* snails

Introduction:

Certain simultaneous hermaphrodites, such as the snail species *Physa acuta*, while preferentially outcrossing, are capable of self-fertilization. While self-reproduction allows an organism to transmit more copies of its genes to offspring, in many cases, inbreeding depression, the relative fitness depression of inbred offspring compared to outbred offspring, is severe enough to where outcrossing is a better strategy. A model to determine optimum delay for self-reproduction under these circumstances is already established, but fails to account for certain organisms' capabilities to reduce their mortality by responding to predators. In this poster, we discuss the theoretical implications of our adaptation to the established model, including expected life history scenarios, differences between the models, and the predicted existence of hysteresis, the lack of reversibility as a parameter is varied. Of particular note is a greatly increased delay before selfing in response to increased predation rate and inbreeding depression.

Model:

In previous research, Tsitrone et al. constructed a mathematical model for optimal mating strategies in simultaneous hermaphrodites, using *Physa acuta* for testing.[1 2] The basic model for this is:

$$R_0 = \int_0^\infty \left[\int_u^\infty l(x)b(x)dx, \right] f(u)du,$$

where $l(x)$ is the probability of surviving to day x, $b(x)$ is the rate of gene transmission on day x, and $f(u)$ is the probability of meeting a mate on day u. We modified this model by replacing its constant mortality rate with a piecewise linear mortality function, which emulates an organism's use of defenses to counter the effects of predation. Since certain defenses, such as size, also increase fecundity, we define k_s as those defenses which also serve to increase fecundity and k_d as other defenses, which solely reduce mortality. The investment into these two defenses is represented by r, the amount of resources allocated to growth.

Parameters:

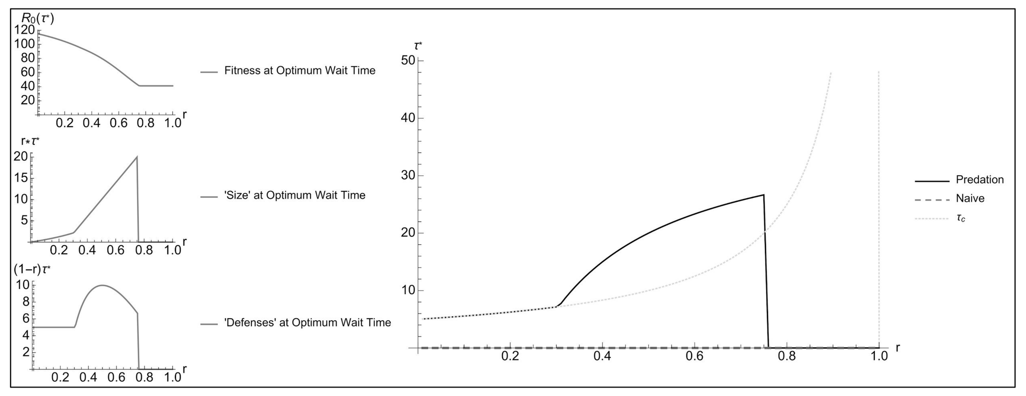
c:	Baseline Reproduction
m_0:	Baseline Mortality
k_r:	Resource Allocation Efficiency
k_s:	Size Defense Efficiency
k_d:	Other Defense Efficiency
e_m:	Mate Encounter Rate
e_p:	Predator Encounter Rate
ϵ:	Predator Success Rate
δ:	Inbreeding Depression
r:	Resources Allocated to Growth

Results:

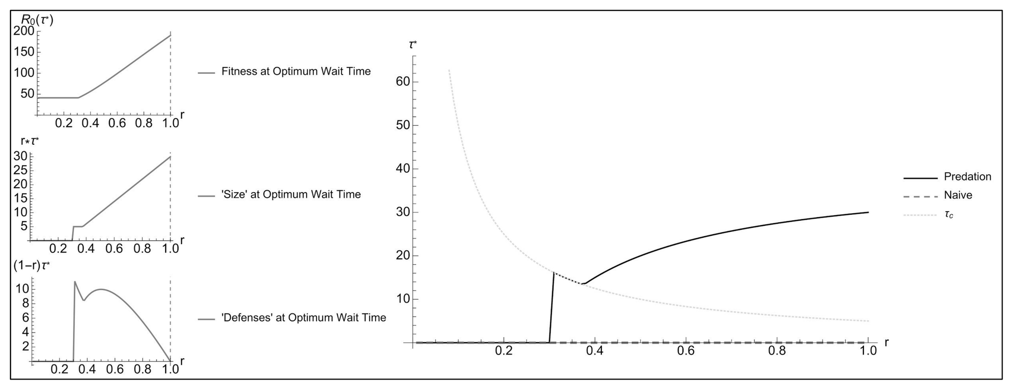
Our recent work on the model has been focused on r, the fraction of resources invested in size. While other parameters, such as predation rate or mate encounter rate, represent environmental factors, r represents various behavioral and physiological attributes of the organisms themselves, and, as such, can be selected for and optimized. Unfortunately, as our outer integrals cannot be analytically solved, we cannot analytically optimize r, and must instead use numerical optimization.

The figures below present r sweeps in different environmental scenarios. The rightmost figure in each set plots the optimal wait time, τ^*, vs. r. The figures down the left hand side plot various values based upon to τ^*, each of which can be measured experimentally. From top to bottom, they are: fitness, size, and defenses. It is important to note that the size and defense plots are not in proper units, but are merely relative quantities, as we have yet to decide upon a proper conversion rate.

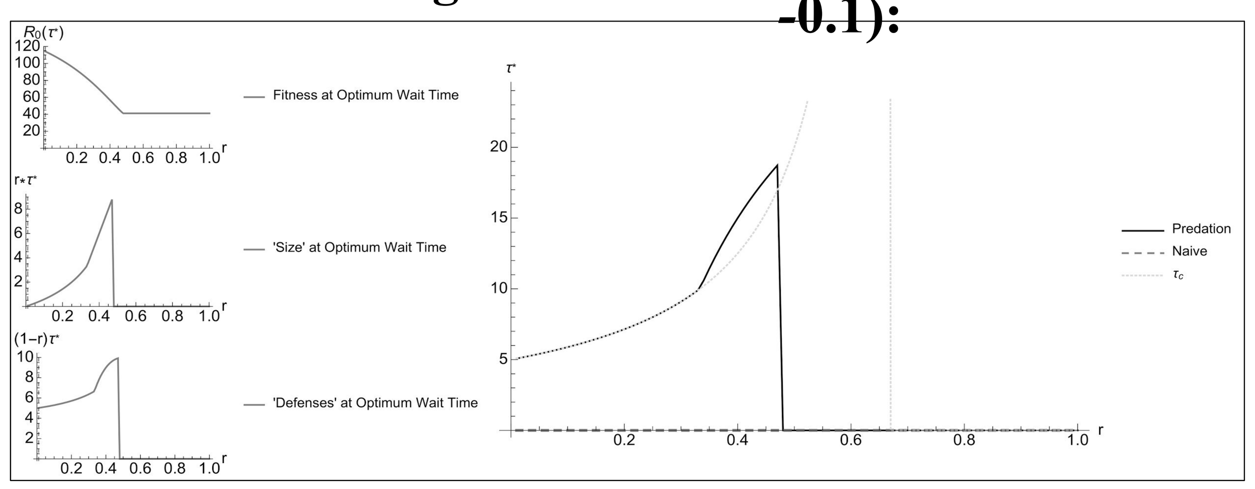
Defense-Attacking Predator: ($k_d = 0.2$, $k_s = 0$):



Low-Size Preferential Predator: ($k_d = 0$, $k_s = 0.2$):

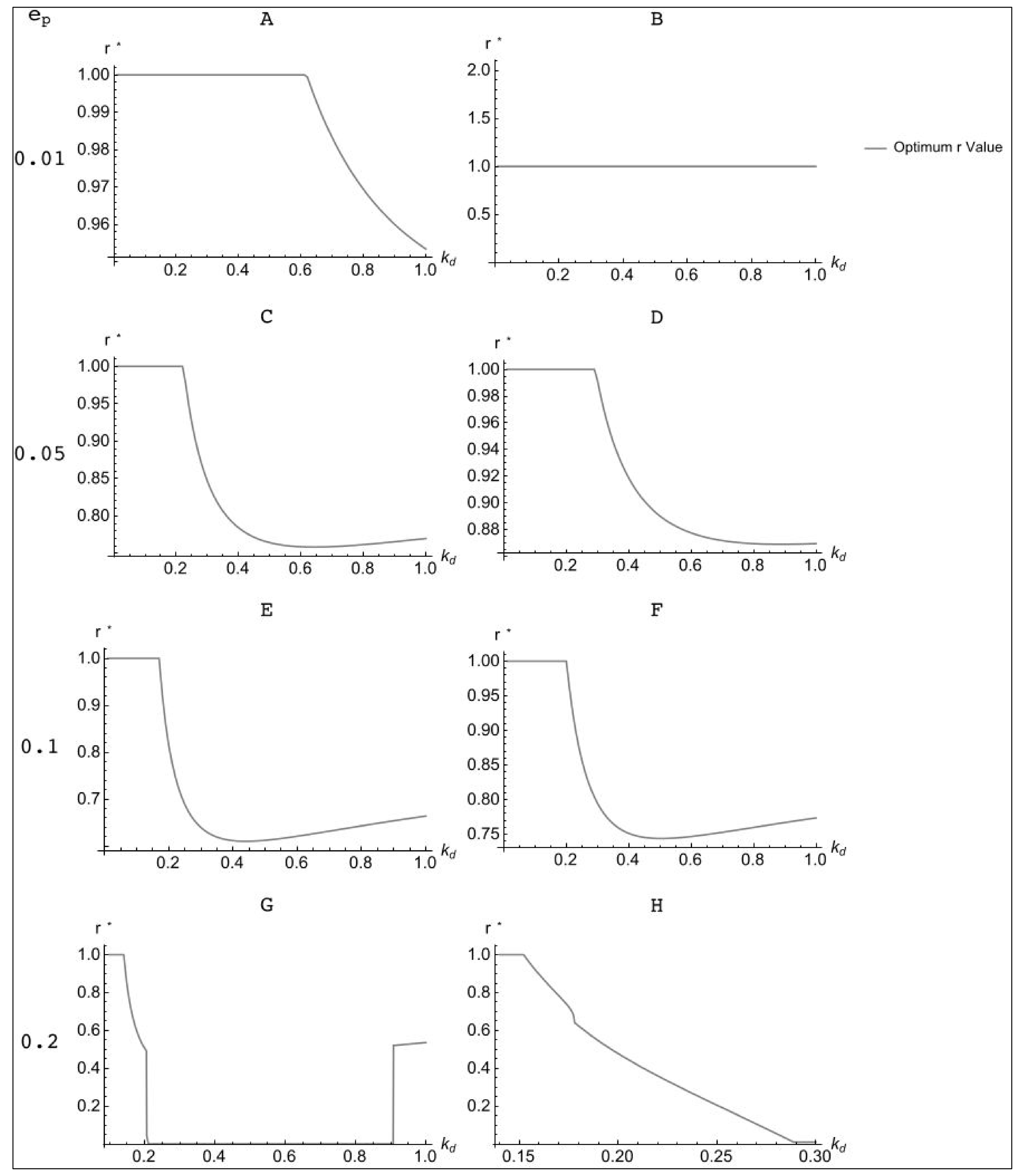


High-Size Preferential, Defense-Attacking Predator ($k_d = 0.2$, $k_s = -0.1$):



In each of the above plots:
$c = 18$, $m_0 = 0.05$, $\epsilon = 1$, $e_p = 0.2$, $e_m = 0.1$, $k_r = 0.1$, $\delta = 0.8$

Optimal *r* Parameter Sweeps:



In the above sweeps: $c = 18$, $m_0 = 0.05$, $\epsilon = 1$, $k_s = 0.1$, $k_r = 0.01$

In A: $e_m = 0.1$, $\delta = 0.8$ In B: $e_m = 0.01$, $\delta = 0.75$

Above, we present plots of optimal r values. As r is primarily a defensive parameter, it relates most closely to the other defensive parameters, k_d and k_s, and the predation parameters, e_p and ϵ. As such, each above plot is of r^* vs k_d, and e_p is larger for lower plots. Note that the plots are most certainly *not* to scale, so the differences between plots are more significant than they may appear graphically.

The sudden jumps in the bottom plots suggest that r^* might have multiple optimal strategies, any of which may be the global maximum. We also noted this phenomena in τ^*.

Acknowledgments
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Literature Cited

1. Tsitrone, A., Duperron, A., & David, P. 2003. Delayed selfing as an optimal mating strategy in preferentially outcrossing species: theoretical analysis of the optimal age at first reproduction in relation to mate availability. American Naturalist, 162, 318–331.
2. Tsitrone, A., Jarne, P., & David, P. 2003. Delayed selfing and resource reallocations in relation to mate availability in the freshwater snail *Physa acuta*. American Naturalist, 162, 474–488.