McKibben Webster – Chapter 5 Partial Solutions and Hints

5.2.1 Yes, a classical solution is a mild solution because you can simply integrate both sides of (NON-CP). The converse, however, is false because a continuous function need not be differentiable.

5.3.1 Line 3: Product rule for matrix functions

Line 4: Equivalent functions have the same antiderivative

Line 5: Fundamental Theorem of Calculus

Lines 6 – 7: Standard matrix operations

Line 8: Linearity of the integral

5.3.2 The mapping $t \mapsto \int_0^t g(s)ds$ is differentiable whenever $g(s)$ is continuous. Why? So what?

5.3.3 (i) $U(t) = \begin{bmatrix} -\frac{3}{4}e^{-2t} + \frac{3}{4} - \frac{5}{4}te^{-2t} - \frac{1}{4}t \\ \frac{5}{4}e^{-2t} - \frac{1}{4} + \frac{1}{2}t \end{bmatrix}$

(iii) $U(t) = \begin{bmatrix} -\sin(3t) + \cos(t) - \frac{3}{2}\sin t \cos^2 t + \frac{\pi}{4}\cos t - \frac{1}{2}t\cos t \\ \sin(3t) + \frac{1}{2}\cos t - \frac{1}{2}\cos^3 t - \frac{\pi}{4}\sin t + \frac{1}{2}t\sin t \end{bmatrix}$

5.3.4 $x(t) = \frac{1}{16}\left(1 + 4te^{4t} - e^{4t}\right)e^{-t}$

5.3.5 (i) Use any discontinuous function for any component of $F(t)$.

(iii) Use $F(t) = \begin{bmatrix} 1/t \\ 0 \\ \vdots \\ 0 \end{bmatrix}$. Such a function is not integrable on any bounded interval containing zero. So, the mild solution does not exist whenever the initial condition is prescribed at $t = 0$.

5.4.1 It goes to zero. Mimic the details of the corresponding result in Chapter 4.