MAT 161—Exam #2A—10/21/14

Name: Solutions

Calculators are NOT allowed. Show all work using correct mathematical notation. Unless otherwise indicated, you may use appropriate short-cut rules for computing derivatives.

1. (15 points) Find the derivative of each of the following functions, and simplify your answers as much as possible.

   (a) $f(x) = \dfrac{4}{x^3}$

   $$f'(x) = \frac{-12}{x^4}$$

   (b) $g(x) = x^2 \ln x$

   $$g'(x) = x^2 \cdot \frac{1}{x} + (\ln x) \cdot 2x$$
   $$= x + 2x \ln x$$

   (c) $h(x) = \sin(2x) + \pi^4$

   $$h'(x) = 2\cos(2x)$$

2. (10 points) Find the equation of the tangent line to the curve $y = \sqrt{x}$ at $x = 9$.

$$\frac{dy}{dx} = \frac{1}{2}x^{-1/2}$$
$$= \frac{1}{2\sqrt{x}}$$
$$\Rightarrow \left.\frac{dy}{dx}\right|_{x=9} = \frac{1}{6} \quad \text{slope}$$

Equation:

$$y - 3 = \frac{1}{6}(x - 9)$$
$$\Rightarrow y = \frac{1}{6}x + \frac{3}{2}$$

3. (15 points) Let $f(x) = x^2 + 3x$. State the definition of the derivative in terms of a limit, and use it to calculate $f'(x)$. No credit will be given for short-cut methods.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + 3(x+h) - (x^2 + 3x)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 3x + 3h - x^2 - 3x}{h}$$

$$= \lim_{h \to 0} \frac{2xh + 3h + h^2}{h}$$

$$= \lim_{h \to 0} (2x + 3 + h) = 2x + 3$$

4. (10 points) A particle's position (in meters) after t seconds is given by $s(t) = 4\cos(\pi t)$.

(a) Find the particle's velocity function.

$$v(t) = s'(t) = -4\pi \sin(\pi t)$$

(b) Find the particle's acceleration (the rate of change of velocity) at $t = 2$.

$$a(t) = v'(t) = -4\pi^2 \cos(\pi t)$$

$$\Rightarrow \quad a(2) = -4\pi^2 \ m/s^2$$

5. (25 points) Find $\frac{dy}{dx}$ for each function below. You do not need to simplify your answers, but you must include all necessary parentheses!

(a) $y = e^x \sec x$

$$\frac{dy}{dx} = e^x \cdot \sec x \tan x + (\sec x) \cdot e^x$$

(b) $y = \dfrac{\sin^{-1} x}{x^3 + 7}$

$$\frac{dy}{dx} = \frac{(x^3+7) \cdot \frac{1}{\sqrt{1-x^2}} - (\sin^{-1} x) \cdot 3x^2}{(x^3+7)^2}$$

(c) $y = x^2\, 7^{\tan x}$

$$\frac{dy}{dx} = x^2 \cdot 7^{\tan x} (\ln 7) \sec^2 x + 7^{\tan x} \cdot 2x$$

(d) $y = \sin^9(e^{5x})$

$$\frac{dy}{dx} = 9 \sin^8(e^{5x}) \cdot \cos(e^{5x}) \cdot 5e^{5x}$$

(e) $y = (\ln(\ln x))^4$

$$\frac{dy}{dx} = 4(\ln(\ln x))^3 \cdot \frac{1}{\ln x} \cdot \frac{1}{x}$$

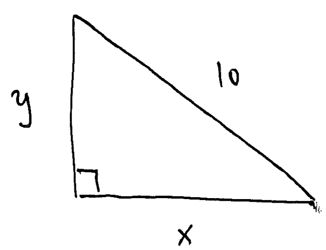
6. (12 points) Find $\frac{dy}{dx}$ for the curve $xy^3 + 5\sin y = 12$.

$$x \cdot 3y^2 \frac{dy}{dx} + y^3 \cdot 1 + 5(\cos y) \frac{dy}{dx} = 0$$

$$\Rightarrow \quad (3xy^2 + 5\cos y) \frac{dy}{dx} = -y^3$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-y^3}{3xy^2 + 5\cos y}$$

7. (13 points) The bottom of a 10-foot ladder slides away from a wall at a rate of 3 ft/sec. How fast is the top of the ladder sliding down the wall when it is 6 feet above the floor?



$$x^2 + y^2 = 100$$

$$\Rightarrow \quad 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

Given : $\frac{dx}{dt} = 3$

Want $\frac{dy}{dt}$ when

$y = 6$, $x = 8$

At the given instant,

$$2 \cdot 8 \cdot 3 + 2 \cdot 6 \cdot \frac{dy}{dt} = 0$$

$$\Rightarrow \quad \frac{dy}{dt} = -4$$

So the top is sliding down at a rate of 4 ft/sec