MAT 161—Exam #3—6/18/15

Name: Solutions

Calculators are NOT allowed. Show all work using correct mathematical notation.

1. (20 points) Consider the function $f(x) = x^4 + 4x^3 + 1$.

   (a) Determine the intervals on which f is increasing/decreasing.

   (b) Determine the intervals on which f is concave up/concave down.

   (c) Sketch a graph of the function, clearly labeling the coordinates of all intercepts, local extrema, and inflection points.

a) $f'(x) = 4x^3 + 12x^2$

$= 4x^2(x+3)$

f': $-$ | -3 | $+$ | 0 | $+$

f: $\searrow$ -3 $\nearrow$ 0 $\nearrow$

-3: min

decreasing on $(-\infty, -3)$, increasing on $(-3, 0) \cup (0, \infty)$

b) $f''(x) = 12x^2 + 24x$

$= 12x(x+2)$

f'': $+$ | -2 | $-$ | 0 | $+$

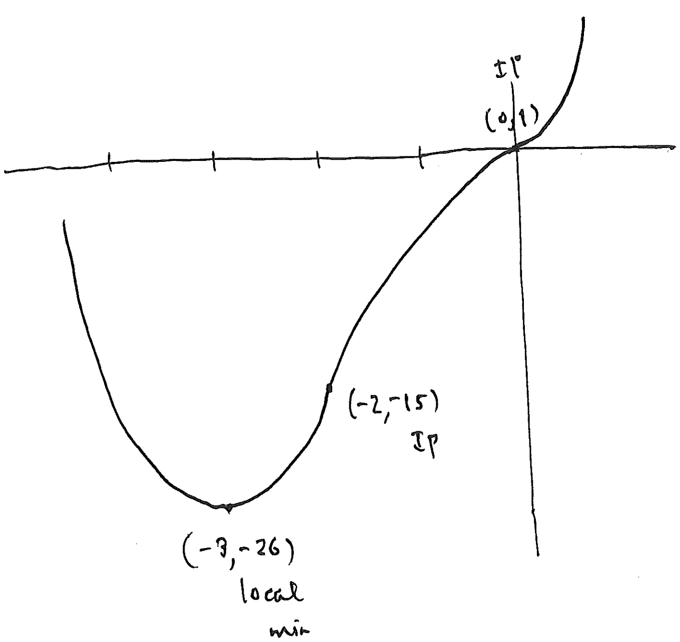
f: $\cup$ -2 $\cap$ 0 $\cup$

-2: IP, 0: IP

concave up on $(-\infty, -2) \cup (0, \infty)$, concave down on $(-2, 0)$

c) Summary:

-3 -2 0



2. (15 points) Evaluate each of the following limits. Show all work using correct notation!

(a) $\lim_{x\to\infty} \frac{\ln(5x+2)}{\ln(4x+3)}$ "$\frac{\infty}{\infty}$"

$$\overset{}{\underset{L'H}{=}} \lim_{x\to\infty} \frac{\frac{5}{5x+2}}{\frac{4}{4x+3}}$$

$$= \lim_{x\to\infty} \frac{20x+15}{20x+8} \underset{L'H}{=} 1$$

(b) $\lim_{x\to 0} \frac{e^{3x}-1-3x}{1-\cos 5x}$ "$\frac{0}{0}$"

$$\underset{L'H}{=} \lim_{x\to 0} \frac{3e^{3x}-3}{5\sin 5x}$$

$$\underset{L'H}{=} \lim_{x\to 0} \frac{9e^{3x}}{25\cos 5x}$$

$$= \frac{9}{25}$$

3. (10 points) Find the absolute maximum and minimum values of $f(x) = x + \frac{1}{x}$ on the interval $[\frac{1}{2}, 3]$.

$$f'(x) = 1 - \frac{1}{x^2} = 0 \Rightarrow x = \pm 1$$

$f(1/2) = 1/2 + 2 = 5/2$

$f(1) = 1 + 1 = 2$ MIN

$f(3) = 3 + 1/3 = 10/3$ MAX

4. (15 points) Consider the function $f(x) = \ln x$.

(a) Find the linearization $L(x)$ of $f(x)$ at $a = 1$.

$$f'(x) = \frac{1}{x} \Rightarrow f'(1) = 1$$

$$L(x) = f(1) + f'(1)(x-1)$$
$$= 0 + 1(x-1)$$
$$= x - 1$$

(b) Use the linearization from part (a) to give an estimate for $\ln(0.93)$.

$$\ln(0.93) \approx L(0.93)$$
$$= 0.93 - 1$$
$$= -0.07$$

5. (15 points) Evaluate each of the following indefinite integrals.

(a) $\int (2x^4 + e^{-2x} + 5\sec^2 x + 7)\, dx$

$$= \frac{2}{5}x^5 - \frac{1}{2}e^{-2x} + 5\tan x + 7x + C$$

(b) $\int \left(\sqrt{x} + \frac{3}{x^2} - \frac{6}{x} + 3\sin 5x\right) dx$

$$= \frac{2}{3}x^{3/2} - \frac{3}{x} - 6\ln|x| - \frac{3}{5}\cos 5x + C$$

6. (10 points) Find the interval(s) on which the function $f(x) = \dfrac{e^{2x}}{x+3}$ is increasing.

$$f'(x) = \frac{(x+3)\cdot 2e^{2x} - e^{2x}\cdot 1}{(x+3)^2}$$

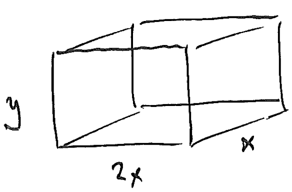
$$= \frac{e^{2x}(2x+5)}{(x+3)^2}$$

f': $-$ $\quad -3 \quad$ $-$ $\quad -5/2 \quad$ $+$

f: $\searrow$ $\quad -3 \quad$ $\searrow$ $\quad -5/2 \quad$ $\nearrow$

increasing on $(-5/2, \infty)$

7. (15 points) A rectangular storage bin with no top is to have volume 10 cubic meters. The length of its base is twice its width. Material for the base costs \$10 per square meter, and material for the sides costs \$6 per square meter. Find the dimensions of the cheapest such container.



Minimize $C = 10\cdot 2x^2 + 6(4xy + 2xy)$

$= 20x^2 + 36xy$

Subject to $2x^2y = 10 \Rightarrow y = \dfrac{5}{x^2}$

Thus $C(x) = 20x^2 + 36x\left(\dfrac{5}{x^2}\right)$

$= 20x^2 + \dfrac{180}{x}$

$C'(x) = 40x - \dfrac{180}{x^2} = 0$

$\Rightarrow 40x^3 = 180$

$\Rightarrow x = (9/2)^{1/3}, \quad y = \dfrac{5}{(9/2)^{2/3}}$

$C''(x) = 40 + \dfrac{360}{x^3} > 0$ for $x > 0$

So C is concave up and hence this solution gives a minimum.