MAT 162—Exam #3—11/22/11

Name: _____

Show all work using correct mathematical notation. Calculators are not permitted.

1. (12 points) Find the limit of each of the following sequences.

    (a) $a_n = \dfrac{e^{3n} + 4}{e^{3n+1} + 5}$

    (b) $a_n = \ln(5n^2 + 1) - \ln(n^2 + 3n + 2)$

2. (13 points) In each case, find the sum of the series or show that the series diverges.

    (a) $\displaystyle\sum_{n=0}^{\infty} \frac{5}{3^n}$

    (b) $\displaystyle\sum_{n=4}^{\infty} \left(\cos\left(\frac{\pi}{n}\right) - \cos\left(\frac{\pi}{n+1}\right) \right)$

3. (25 points) Decide whether each series is convergent or divergent, and justify your answers using appropriate tests. You must give coherent arguments to receive credit.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{n^2+1}{n^3-n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{7^{2n}}{\sqrt{n!}}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{3/2}}$

4. (10 points) Let $a_n = \dfrac{n+7}{3n+5}$. Evaluate

(a) $\lim\limits_{n\to\infty} a_n$

(b) $\sum\limits_{n=1}^{\infty} a_n$

5. (15 points) Consider the series $S = \sum\limits_{n=3}^{\infty} \dfrac{(-1)^{n-1}}{\ln(\ln n)}$.

(a) Show that the series converges conditionally. You must give a clear and complete argument, citing any appropriate tests.

(b) Let $S_N = \sum\limits_{n=3}^{N} \dfrac{(-1)^{n-1}}{\ln(\ln n)}$. How large must N be to ensure that $|S - S_N| < \frac{1}{10}$?

6. (10 points) Decide whether each statement is true or false. If a statement is false, give an example to show why.

(a) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

(b) If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} |a_n|$ converges.

7. (15 points) Find the radius and interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(x+2)^n}{\sqrt{n}\, 5^n}.$$

Justify your conclusions by citing appropriate tests.