# MA 208—Final Exam—12/15/08

Name: ________________________________ Instructor: Parsell

Calculators are permitted, but you must show all of your work using correct notation.

1. (10 points) Find the angle between the vectors $\mathbf{u} = \mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$ and $\mathbf{v} = 5\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ to the nearest degree.

2. (15 points) Find the area of the triangle determined by the points $P(1, -1, 2)$, $Q(2, 1, 3)$, and $R(1, 2, -1)$.

3. (15 points) Find parametric equations for the line through $(1, 2, 3)$ perpendicular to the plane $4x - y + 7z = 5$, and determine the point where the line intersects the plane.

4. (10 points) The position vector of a particle in two dimensions at time $t$ is given by the formula $\mathbf{r}(t) = (t^3 + 1)\mathbf{i} + (20 - 4\sqrt{t + 3})\mathbf{j}$.

   (a) Determine the particle's speed at the instant when $t = 1$.

   (b) Find the $x$-coordinate of the particle's position at the instant when the $y$-coordinate is zero.

5. (10 points) Evaluate $\lim\limits_{\substack{(x,y)\to(3,3) \\ x\neq y}} \dfrac{x^2 - xy}{x^4 - y^4}$ or prove that it doesn't exist.

6. (15 points) Consider the function $f(x, y, z) = z - \ln(x^2 + y^2)$.

(a) Find the derivative of $f$ at the point $(1, 2, 3)$ in the direction of $\mathbf{A} = \mathbf{i} - \mathbf{j} + \mathbf{k}$.

(b) Find an equation for the tangent plane to the level surface of $f$ through the point $(1, 2, 3)$.

7. (15 points) Determine the location of all local maxima, local minima, and saddle points of the function

$$f(x, y) = 2x^3 + 3xy + 2y^3.$$

8. (10 points) Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = x + y^2$ on the circle $x^2 + y^2 = 1$.

9. (12 points) Evaluate $\int_0^8 \int_{\sqrt[3]{x}}^2 \frac{dy\,dx}{y^4+1}$ by reversing the order of integration. Be sure to include a sketch of the region of integration.

10. (13 points) Find the average height of the hemisphere $z = \sqrt{25 - x^2 - y^2}$ above the disk $x^2 + y^2 \leq 25$ in the $xy$-plane.

11. (10 points) A solid of density $\delta(x, y, z) = x + y + 5$ and total mass $M$ occupies the region cut from the cylinder $x^2 + y^2 = 4$ by the plane $z = 0$ and the plane $x + z = 3$. Set up (but do not evaluate) an integral that gives the $z$-coordinate of the object's center of mass.

12. (15 points) Find the volume of the ice cream cone bounded above by the sphere $x^2 + y^2 + z^2 = 4$ and below by the cone $z = \sqrt{x^2 + y^2}$.

13. (12 points) Find the mass of a wire lying along the helix $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + t^2\mathbf{k}$, $0 \leq t \leq 1$, if the density is $\delta(t) = 2t$.

14. (13 points) Find the work done by the force $\mathbf{F} = z^2\mathbf{i} + x\mathbf{j} + y\mathbf{k}$ over the curve defined by $\mathbf{r}(t) = 4t\mathbf{i} + t\mathbf{j} + e^t\mathbf{k}$, $0 \leq t \leq 1$.

15. (10 points) Find a potential function for the conservative vector field

$$\mathbf{F} = (2xz\sin y + e^x \cos z)\mathbf{i} + (3y^2 + x^2 z\cos y)\mathbf{j} + (x^2 \sin y - e^x \sin z + 4)\mathbf{k}.$$

16. (15 points) Suppose that $\mathbf{F} = x^3y^2\mathbf{i} + (x - 2y)\mathbf{j}$ represents the velocity field of a fluid, and let $C$ be the boundary of the region enclosed by the curve $y = x^2$ and the lines $y = 0$ and $x = 1$.

(a) Find the counterclockwise circulation of $\mathbf{F}$ around $C$.

(b) Find the outward flux of $\mathbf{F}$ across $C$.