MAT 261—Exam #2—10/16/14

Name: _____

Calculators are not permitted. Show all of your work using correct mathematical notation.

1. (10 points) Find and sketch the domain of the function $f(x, y) = \sqrt{y} + \sqrt{4 - x^2 - y^2}$.

2. (15 points) Let $f(x, y, z) = \dfrac{x^4 \ln z}{y^5} + e^{xy+yz^3} \tan(z^7)$. Calculate f_x, f_y, and f_z.

3. (10 points) Let $f(x,y) = \dfrac{x^2 y}{(x+y)^3}$. Show that $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ does not exist.

4. (15 points) Consider the function $f(x,y,z) = \frac{1}{4}x^2y^3z^5$.

   (a) Find the directional derivative of f at the point $(1,2,1)$ in the direction of $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$.

   (b) Find the maximum value of the directional derivative of f at the point $(1,2,1)$.

   (c) Find the equation of the tangent plane to the level surface $f(x,y,z) = 2$ at the point $(1,2,1)$.

5. (10 points) Find the linearization of the function $f(x, y) = x^2 \cos y$ at the point $(3, \pi/3)$.

6. (15 points) Let $w = \dfrac{4}{2x + 3y}$, where $x = r\cos\theta$ and $y = r\sin\theta$. Calculate $\partial w/\partial\theta$ at the point $(r, \theta) = (2, 3\pi/4)$,

7. (13 points) Find the coordinates of all local maxima, local minima, and saddle points of the function $f(x, y) = 2x^2 + 3xy + 4y^2 - 5x + 2y$.

8. (12 points) Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = 3x - y$ on the circle $x^2 + y^2 = 40$.