MAT 261—Exam #2A—3/13/14

Name: ______________________________

Calculators are not permitted. Show all of your work using correct mathematical notation.

1. (25 points) Consider the function $f(x, y) = \ln(x^2 - y)$.

   (a) Sketch the domain of f.

   (b) Find the equation of the level curve of f passing through the point $(2, 3)$, and sketch its graph.

   (c) Find the average rate of change of f with respect to y from $(2, 1)$ to $(2, 3)$.

   (d) Find the gradient of f at the point $(2, 1)$.

   (e) Find the instantaneous rate of change of f at the point $(2, 1)$ in the direction $\mathbf{j}$.

2. (15 points) Let $f(x, y, z) = z^3 \cos(xy^2) + e^{xyz} \tan z$. Calculate f_x, f_y, and f_z.

3. (10 points) Find the linearization of the function $f(x, y) = \sqrt{x^2 + y^4}$ at the point $(3, 2)$.

4. (30 points) Consider the function $f(x, y) = x^2 + y^2 - xy + x$.

(a) Find the maximum value of the directional derivative of f at the point $(3, 5)$.

(b) Find the directional derivative of f at the point $(3, 5)$ in the direction of $\mathbf{v} = 3\mathbf{i} - \mathbf{j}$.

(c) Find the coordinates of all local maxima, local minima, and saddle points of f.

(d) If $x = \sin 2t$ and $y = 2e^{3t}$, calculate $\dfrac{df}{dt}$ when $t = 0$.

5. (10 points) Let $f(x,y) = \dfrac{x^2y}{x^4+y^2}$. Show that $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ does not exist.

   Hint: Consider a parabolic path of approach.

6. (10 points) Use Lagrange multipliers to find the point on the line $4x - 6y = 25$ where the function $f(x,y) = x^2 + 2y^2$ has its minimum value.